An analytical criterion for stability boundaries of non-autonomous systems based on Melnikov's method

Susuki, Yoshihiko; Hikihara Takashi

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An Analytical Criterion for Stability Boundaries of Non-Autonomous Systems Based on Melnikov's Method*

Yoshihiko Susuki† and Takashi Hikihara†

メルニコフの方法に基づく非自励系の安定境界に関するある解析的条件*

薄　良彦†・引原　隆士†

This paper proposes an analytical criterion for stability boundaries of non-autonomous systems. The criterion can analytically enlarge the conservative stability limits obtained by the classical Lyapunov's direct method almost up to the exact stability boundaries even for non-autonomous systems. It is based on the Melnikov's method which estimates homoclinic intersections in the dynamical systems theory. The definition of the criterion has strong advantages in its easy and quick estimation of the stability, compared with the numerical integration of the non-autonomous systems. The effectiveness is confirmed in its application to an electric power system with dc transmission under periodic swing.

1. Introduction

This paper proposes an analytical criterion for stability boundaries of non-autonomous systems.

In terms of safety design on engineering systems, it is important to estimate stability regions and their boundaries in which systems can be normally operated in spite of external small disturbance. This leads to the necessity to decide the basin boundaries of asymptotic stable equilibrium points and periodic solutions. Generally speaking, the stability boundaries of the autonomous systems have been evaluated by the following two methods: Lyapunov function approach [1,2] and dynamical systems theory [3–6]. Their methods can clarify the stability boundaries in the autonomous systems. However, if the systems are non-autonomous, their approaches are hardly applied. The reason is that Lyapunov function approach can not substantially deal with the periodic solutions of the non-autonomous systems, and in the second one, discrete dynamical systems derived from the non -autonomous systems can not be analytically represented. For the present, the evaluation on the stability regions of the non-autonomous systems depends on the calculation of each solution in the initial value plane by numerical integrations [7]. It is hence significant to establish the analytical criterion for the stability boundaries of the non-autonomous systems.

In this paper, we study the stability boundaries, which are the analogue of the separatrices in the Hamiltonian systems, in perturbed Hamiltonian systems. The problem formulation is common to the original study by Melnikov, well-known as Melnikov's method [8]. The Melnikov's method provides us with a signed distance between the stable and unstable manifolds in the perturbed systems based on the separatrices which correspond to the stability limits obtained by the direct method. This suggests the possibility to measure the distance between the separatrices and the stable manifolds which coincide with the stability boundaries in the perturbed systems under certain conditions. In the following discussion, the analytical criterion for the stability boundaries is proposed through a modification of the separatrices based on the Melnikov's method [9].

In addition, as a practical application of the criterion, we discuss an electric power system with dc transmission under periodic swing. Here, a swing equation with periodic force derived in Ref. [10–12] is considered.

The organization of the paper is as follows; The basic system considered in this paper is introduced

* **2001 年 11 月 30 日
† 京都大学大学院 工学研究科 電気工学専攻
Department of Electrical Engineering, Kyoto University; Yoshida-Honmachi, Sakyo-ku, Kyoto, 606-8501, JAPAN
Key Words: analytical criterion, stability boundary, non-autonomous system, Melnikov's method, electric power system with dc transmission.

in Section 2. In Section 3, the outline of the Melnikov's method is given. Section 4 provides us with the analytical criterion for the stability boundaries. In Section 5, we discuss the application of our proposed criterion to an electric power system with dc transmission under periodic swing.

2. Basic System and Preliminaries

In this paper, the second-order perturbed Hamiltonian system is considered. The system is given by

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\partial}{\partial y}H(x,y) + \varepsilon g_1(x,y,t), \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -\dfrac{\partial}{\partial x}H(x,y) + \varepsilon g_2(x,y,t), \end{cases} \tag{1}$$

where $(x,y)\in\mathbf{R}\times\mathbf{R}$, and $H(x,y)$ represents the Hamiltonian, ε the small positive parameter and $\varepsilon g_i(x,y,t)$ $(i=1,2)$ the perturbation terms. Here, the right-hand side of Eq. (1) is assumed to be tractable in the region we are interested in. In addition, the perturbation terms are assumed to have a periodicity on t with period $T=2\pi/\Omega$. The vector formula of the system (1) is represented as follows:

$$\frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t} = \mathrm{J}\mathrm{D}H(\boldsymbol{q})+\varepsilon\boldsymbol{g}(\boldsymbol{q},t), \tag{2}$$

where $\boldsymbol{q}\triangleq(x,y)^{\mathrm{T}}$, $\boldsymbol{g}(\boldsymbol{q},t)\triangleq(g_1(x,y,t),g_2(x,y,t))^{\mathrm{T}}$,

$$\mathsf{J}\triangleq\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{3}$$

and

$$\mathrm{D}H(\boldsymbol{q})\triangleq\left(\frac{\partial}{\partial x}H(x,y),\frac{\partial}{\partial y}H(x,y)\right)^{\mathrm{T}}. \tag{4}$$

T represents the transpose operation of vectors.

When ε is equal to zero, the system (1) becomes a Hamiltonian system as follows:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\partial}{\partial y}H(x,y), \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -\dfrac{\partial}{\partial x}H(x,y), \end{cases} \tag{5}$$

or, in a vector form,

$$\frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t} = \mathrm{J}\mathrm{D}H(\boldsymbol{q}). \tag{6}$$

Here, the Hamiltonian system (5) holds the following assumption:

Assumption 1 In the Hamiltonian system (5), there exists at least a hyperbolic equilibrium (saddle) point $\boldsymbol{p}_0$ connected to itself by a separatrix (homoclinic orbit) $\boldsymbol{q}_0(t)$.

Based on the above assumption, Fig. 1 shows the schematic phase structure of the Hamiltonian system which we consider in this paper. In the following discussion, we study the stability boundary, which is the analogue of the separatrix in the Hamiltonian system (6), in the perturbed Hamiltonian system (2).

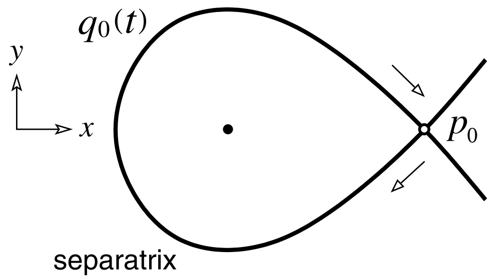


Fig. 1 Schematic phase structure of the Hamiltonian system.

3. Outline of Melnikov's Perturbation Method

In this section, the outline of the Melnikov's perturbation method is given. The Melnikov's method provides us with a signed distance between the stable and unstable manifolds in the perturbed Hamiltonian system based on the separatrix in the Hamiltonian system [8,13,14]. In this section, the distance between the separatrix and the stable manifold is analytically derived based on the Melnikov's method. It is an expansion of the well-known derivation of the Melnikov's method [13,14].

If the perturbation parameter ε is sufficiently small, the following lemmas present the information about the phase structure of the perturbed Hamiltonian system (2).

[**Lemma 1**] For sufficiently small ε, the Hamiltonian system (2) has a unique hyperbolic periodic solution of the saddle type $\boldsymbol{\gamma}_\varepsilon(t)=\boldsymbol{p}_0+\mathcal{O}(\varepsilon)$. Correspondingly, the stroboscopic observation at a phase ϕ_0 has an unique hyperbolic fixed point of the saddle type $\boldsymbol{p}_\varepsilon=\boldsymbol{p}_0+\mathcal{O}(\varepsilon)$.

[**Lemma 2**] The local invariant manifolds of the fixed point on a stroboscopic phase ϕ_0 are C^r close to those of the the equilibrium point $\boldsymbol{p}_0$. C^r here stands for the r times differentiable.

(Proof) These proofs are given in Ref. [13,14]. **

The global stable and unstable manifolds of the fixed point $\boldsymbol{p}_\varepsilon$ can be obtained from the local stable and unstable manifolds of the fixed point by time evolution of the perturbed system (2). In addition, it should be noted that for the tractable properties of the system (2), our analysis can be restricted to an $\mathcal{O}(\varepsilon)$ neighborhood of the separatrix.

Based on **Lemma 1** and **Lemma 2**, Fig. 2 shows the schematic phase structure of the perturbed system (2) under the stroboscopic observation for sufficiently small ε. In the figure, $\boldsymbol{q}_0(-t_0)$ denotes a point

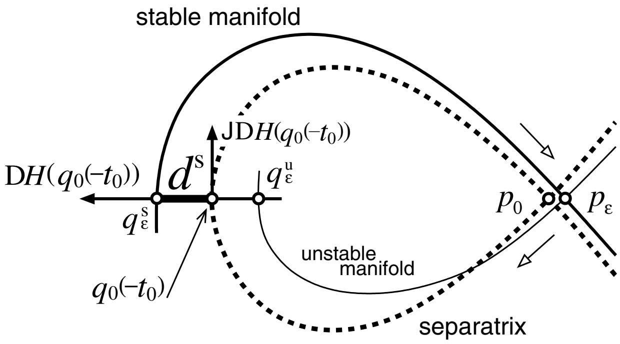


Fig. 2 Schematic phase structure of the perturbed Hamiltonian system under the stroboscopic observation for sufficiently small ε.

on the separatrix as a parameter $t_0 \in \mathbf{R}$, $\mathrm{J}\mathrm{D}H(\boldsymbol{q}_0(-t_0))$ the tangent vector at the point $\boldsymbol{q}_0(-t_0)$ and $\mathrm{D}H(\boldsymbol{q}_0(-t_0))$ the normal vector at the point $\boldsymbol{q}_0(-t_0)$. Moreover, in Fig. 2, $\boldsymbol{q}_\varepsilon^{\mathrm{s}}$ represents the intersection of the normal vector $\mathrm{D}H(\boldsymbol{q}_0(-t_0))$ and the stable manifold of the saddle point $\boldsymbol{p}_\varepsilon$, and $\boldsymbol{q}_\varepsilon^{\mathrm{u}}$ the intersection of the normal vector $\mathrm{D}H(\boldsymbol{q}_0(-t_0))$ and the unstable manifold of $\boldsymbol{p}_\varepsilon$.

The Melnikov's method provides us with the signed distance between the points $\boldsymbol{q}_\varepsilon^{\mathrm{s}}$ and $\boldsymbol{q}_\varepsilon^{\mathrm{u}}$ on the assumption that ε is sufficiently small. The distance $d(\boldsymbol{q}_0(-t_0),\phi_0,\varepsilon)$ is easily induced as follows:

$$d(\boldsymbol{q}_0(-t_0),\phi_0,\varepsilon) \triangleq \frac{\mathrm{D}H(\boldsymbol{q}_0(-t_0))\cdot(\boldsymbol{q}_\varepsilon^{\mathrm{s}}-\boldsymbol{q}_\varepsilon^{\mathrm{u}})}{||\mathrm{D}H(\boldsymbol{q}_0(-t_0))||}, \tag{7}$$

where ϕ_0 represents the stroboscopic phase and $||\cdot||$ the Euclidean norm.

In the estimation of stability boundary, the distance between the point $\boldsymbol{q}_0(-t_0)$ on the separatrix and the point $\boldsymbol{q}_\varepsilon^{\mathrm{s}}$ on the stable manifold is an important value. It is thus defined by $d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0,\varepsilon)$:

$$d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0,\varepsilon) \triangleq \frac{\mathrm{D}H(\boldsymbol{q}_0(-t_0))\cdot(\boldsymbol{q}_\varepsilon^{\mathrm{s}}-\boldsymbol{q}_0(-t_0))}{||\mathrm{D}H(\boldsymbol{q}_0(-t_0))||}. \tag{8}$$

It should be noted that for $\varepsilon=0$ the point $\boldsymbol{q}_0(-t_0)$ corresponds to the point $\boldsymbol{q}_\varepsilon^{\mathrm{s}}$, i.e.

$$d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0,0)=0. \tag{9}$$

Here, Taylor expansion of Eq. (7) around $\varepsilon=0$ gives

$$d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0,\varepsilon)=d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0,0) + \left(\frac{\partial}{\partial\varepsilon}d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0,\varepsilon)\right)\Bigg|_{\varepsilon=0}\varepsilon+\mathcal{O}(\varepsilon^2), \tag{10}$$

where

$$\frac{\partial}{\partial\varepsilon}d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0,\varepsilon) = \frac{1}{||\mathrm{D}H(\boldsymbol{q}_0(-t_0))||}\mathrm{D}H(\boldsymbol{q}_0(-t_0))\cdot\frac{\partial\boldsymbol{q}_\varepsilon^{\mathrm{s}}}{\partial\varepsilon}. \tag{11}$$

The distance $d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0,\varepsilon)$ can be hence described by

$$d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0,\varepsilon)=\frac{\varepsilon\Delta^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0)}{||\mathrm{D}H(\boldsymbol{q}_0(-t_0))||}+\mathcal{O}(\varepsilon^2), \tag{12}$$

where

$$\Delta^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0) \triangleq \mathrm{D}H(\boldsymbol{q}_0(-t_0))\cdot\frac{\partial\boldsymbol{q}_\varepsilon^{\mathrm{s}}}{\partial\varepsilon}\Bigg|_{\varepsilon=0}. \tag{13}$$

On the standpoints of practical application, it is desirable that the distance $d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0,\varepsilon)$ can be calculated without the information of the stable manifold $\boldsymbol{q}_\varepsilon^{\mathrm{s}}$ in the perturbed system. This is achieved by utilizing the Melnikov's original technique [8]. Here, the following time-dependent function is defined:

$$\tilde{\Delta}^{\mathrm{s}}(t;\boldsymbol{q}_0(-t_0),\phi_0) \triangleq \mathrm{D}H(\boldsymbol{q}_0(t-t_0))\cdot\boldsymbol{q}_1^{\mathrm{s}}(t), \tag{14}$$

where

$$\boldsymbol{q}_1^{\mathrm{s}}(t) \triangleq \frac{\partial}{\partial\varepsilon}\boldsymbol{q}_\varepsilon^{\mathrm{s}}(t)\Bigg|_{\varepsilon=0}. \tag{15}$$

Here, $\boldsymbol{q}_\varepsilon^{\mathrm{s}}(t)$ denotes the trajectory on the stable manifold and satisfies

$$\boldsymbol{q}_\varepsilon^{\mathrm{s}}(0)=\boldsymbol{q}_\varepsilon^{\mathrm{s}}. \tag{16}$$

The expression $\boldsymbol{q}_0(t-t_0)$ represents the separatrix. Obviously, using Eqs. (13)–(16), the term $\Delta^{\mathrm{s}}(\boldsymbol{q}(-t_0),\phi_0)$ is defined:

$$\Delta^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0) \equiv \tilde{\Delta}^{\mathrm{s}}(0;\boldsymbol{q}_0(-t_0),\phi_0). \tag{17}$$

Differentiating (14) with respect to t, the following formula is given:

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Delta}^{\mathrm{s}}(t;\boldsymbol{q}_0(-t_0),\phi_0)=\left(\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{D}H(\boldsymbol{q}_0(t-t_0))\right)\cdot\boldsymbol{q}_1^{\mathrm{s}}(t) + \mathrm{D}H(\boldsymbol{q}_0(t-t_0))\cdot\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{q}_1^{\mathrm{s}}(t). \tag{18}$$

Here, the following lemma is provided:

[**Lemma 3**] $\boldsymbol{q}_1^{\mathrm{s}}(t)$ satisfies the following formula:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{q}_1^{\mathrm{s}}(t)=\mathrm{J}\mathrm{D}^2H(\boldsymbol{q}_0(t-t_0))\boldsymbol{q}_1^{\mathrm{s}}(t) +\boldsymbol{g}(\boldsymbol{q}_0(t-t_0),\Omega t+\phi_0). \tag{19}$$

(Proof) The proof is given in Appendix. **

From **Lemma 3**, the substitution of Eq. (19) into Eq. (18) results in the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Delta}^{\mathrm{s}}(t;\boldsymbol{q}_0(-t_0),\phi_0)=\left(\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{D}H(\boldsymbol{q}_0(t-t_0))\right)\cdot\boldsymbol{q}_1^{\mathrm{s}}(t) +\mathrm{D}H(\boldsymbol{q}_0(t-t_0))\cdot\mathrm{J}\mathrm{D}^2H(\boldsymbol{q}_0(t-t_0))\boldsymbol{q}_1^{\mathrm{s}}(t) +\mathrm{D}H(\boldsymbol{q}_0(t-t_0))\cdot\boldsymbol{g}(\boldsymbol{q}_0(t-t_0),\Omega t+\phi_0). \tag{20}$$

Additionally, the next lemma is obtained:

[**Lemma 4**] The following relation is satisfied:

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{D}H(\boldsymbol{q}_0(t-t_0))\right)\cdot\boldsymbol{q}_1^{\mathrm{s}}(t)+ \\ \mathrm{D}H(\boldsymbol{q}_0(t-t_0))\cdot\mathrm{JD}^2H(\boldsymbol{q}_0(t-t_0))\boldsymbol{q}_1^{\mathrm{s}}(t)=0. \quad (21)$$

(Proof) See the proof in Ref. [14]. **

Using **Lemma 4**, Eq. (20) can be rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Delta}^{\mathrm{s}}(t;\boldsymbol{q}_0(-t_0),\phi_0)= \\ \mathrm{D}H(\boldsymbol{q}_0(t-t_0))\cdot\boldsymbol{g}(\boldsymbol{q}_0(t-t_0),\Omega t+\phi_0). \quad (22)$$

Integrating $\tilde{\Delta}^{\mathrm{s}}(t;\boldsymbol{q}_0(-t_0),\phi_0)$ from 0 to τ ($\tau>0$), the following formula is reduced:

$$\tilde{\Delta}^{\mathrm{s}}(\tau;\boldsymbol{q}_0(-t_0),\phi_0)-\tilde{\Delta}^{\mathrm{s}}(0;\boldsymbol{q}_0(-t_0),\phi_0)= \\ \int_0^{\tau}\mathrm{D}H(\boldsymbol{q}_0(t-t_0))\cdot\boldsymbol{g}(\boldsymbol{q}_0(t-t_0),\Omega t+\phi_0)\,\mathrm{d}t. \quad (23)$$

Then, the following lemma is naturally obtained:

[**Lemma 5**] For the first term of the left-hand side of Eq. (23), the following limit is given:

$$\lim_{\tau\to+\infty}\tilde{\Delta}^{\mathrm{s}}(\tau;\boldsymbol{q}_0(-t_0),\phi_0)=0. \quad (24)$$

(Proof) The proof is also given in Ref. [14]. **

According to Eq. (17), the term $\Delta^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0)$ is obtained as follows:

$$\begin{aligned}\Delta^{\mathrm{s}}&(\boldsymbol{q}_0(-t_0),\phi_0)\\ &=\tilde{\Delta}^{\mathrm{s}}(0;\boldsymbol{q}_0(-t_0),\phi_0)\\ &=-\int_0^{+\infty}\mathrm{D}H(\boldsymbol{q}_0(t-t_0))\\ &\qquad\cdot\boldsymbol{g}(\boldsymbol{q}_0(t-t_0),\Omega t+\phi_0)\,\mathrm{d}t. \quad (25)\end{aligned}$$

If the transformation $t\to t+t_0$ is applied, the term $\Delta^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0)$ is given by

$$\begin{aligned}\Delta^{\mathrm{s}}&(\boldsymbol{q}_0(-t_0),\phi_0)\\ &=-\int_{-t_0}^{+\infty}\mathrm{D}H(\boldsymbol{q}_0(t))\\ &\qquad\cdot\boldsymbol{g}(\boldsymbol{q}_0(t),\Omega(t+t_0)+\phi_0)\,\mathrm{d}t. \quad (26)\end{aligned}$$

The term $\Delta^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0)$ makes it possible to calculate the distance $d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0)$ between the separatrix and the stable manifold. Here, in order to derive the new criterion in the next section, the obtained properties about the Melnikov's method are summarized as follows:

(1) The distance

$$d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0)=\frac{\varepsilon\Delta^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0)}{||\mathrm{D}H(\boldsymbol{q}_0(-t_0))||}+\mathcal{O}(\varepsilon^2) \quad (27)$$

is a signed measure. The sign makes it possible to grasp the relationship between the separatrix and the stable manifold.

(2) The distance $d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0)$ diverges to infinity as $t_0\to\pm\infty$. This is because the norm of the normal vector $\mathrm{D}H(\boldsymbol{q}_0(-t_0))$ converges to zero as $t_0\to\pm\infty$. This implies that a point $\boldsymbol{q}_0(-t_0)$ in the neighborhood of the assumed saddle point can not be modified by the distance $d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0)$.

(3) When the perturbation is independent on t, the distance between the separatrix and the stable manifold is also derived by the similar discussion. This implies that our proposed criterion can be applied to autonomous systems, in particular, dissipative systems. As a result, the criterion drastically improves the conventional estimation of the stability limits by other analytical methods, for examples, classical Lyapunov's direct methods.

4. Proposed Criterion for the Stability Boundaries

In this section, an analytical criterion for the stability boundaries in the non-autonomous systems is proposed based on the above preliminaries.

4.1 Method for Obtaining the Criterion

From the previous discussion, the method for modification of the separatrix $\boldsymbol{q}_0(-t_0)$ can be proposed as follows:

Proposed Method Each point $\boldsymbol{q}_0(-t_0), t_0\in\mathbf{R}$ on the separatrix is modified by the following formula:

$$\boldsymbol{q}_0'(-t_0)\stackrel{\triangle}{=}\boldsymbol{q}_0(-t_0) \\ +\frac{d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0)}{||\mathrm{D}H(\boldsymbol{q}_0(-t_0))||}\mathrm{D}H(\boldsymbol{q}_0(-t_0)), \quad (28)$$

where $\boldsymbol{q}_0'(-t_0)$ denotes the modified $\boldsymbol{q}_0(-t_0)$, and $d^{\mathrm{s}}(\boldsymbol{q}_0(-t_0),\phi_0)$ is given by Eq. (27).

4.2 Proposed Criterion

Based on the above method in Section 4.1, we defined an analytical criterion for the stability boundaries in the perturbed system (2). Through the proposed method, it is expected that the modified separatrix $\boldsymbol{q}_0'(-t_0)$ is close to the stable manifold. That is, it has a possibility to become the analytical criterion for the stability boundaries of non-autonomous systems under certain conditions. This paper proposes the modified separatrix $\boldsymbol{q}_0'(-t_0)$ as an analytical criterion for the stability boundary in the perturbed system (2).

4.3 Some Problems on Our Proposed Criterion

This section discuss some problems on our proposed criterion. Above Section 4.2 provided us with the analytical criterion for the stability boundaries and the method for the definition of the analytical criterion. It is inevitable to overcome the following conditions in its wide application.

First, the genesis of other attractors possibly happens in the non-autonomous systems. The genesis is often observed in the various non-autonomous systems [5,11,12,15], and is much interested from mathematical point of view. However, in many practical systems, the genesis is avoided by the design of their systems because it depends on the system parameters strongly. The given application in Section 5 is one of the applications to the practical systems.

In addition, it should be noted that this method is applicable only to the system which has sufficiently small perturbation. The reason is that the Melnikov's method presents much information for the perturbed system with sufficiently small ε.

5. Application to Electric Power System with DC Transmission under Periodic Swing

In this section, the proposed criterion is applied to an electric power system with dc transmission under periodic swing.

5.1 Swing Equation with Periodic Forcing

The following discussion performs the numerical simulation of the swing equation with periodic forcing. Our previous studies [10–12] derive the following swing equation to represent the dynamics of the ac/dc system shown in Fig. 3:

$$\begin{cases} H(\delta,\omega) &= \frac{1}{2}\omega^2 - b\cos\delta - \left(p_{\rm m} - p_{\rm e(dc)}\right)\delta, \\ g_1(\delta,\omega,t) &= 0, \\ g_2(\delta,\omega,t) &= -D\omega + a\cos\Omega t, \end{cases} \tag{29}$$

where δ denotes the rotor angle of the generator, ω the rotor speed deviation, b the critical power of the system, $p_{\rm m}$ the mechanical power input to the generator and $p_{\rm e(dc)}$ the active power flow into the dc transmission. D is related to the damping of the system and a the amplitude of the power swing. Ω is the angular frequency of the power swing.

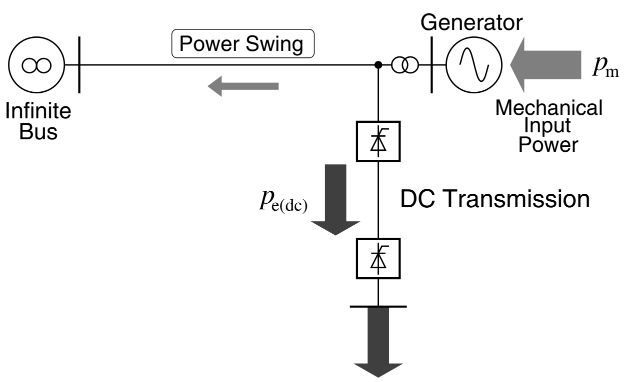


Fig. 3 System configuration of electric power system with dc transmission. The arrows denote the positive direction of power flow.

The swing equation is proposed to analyze the transient stability of the practical system [16]. In this system, control of power swing through the dc power modulation is discussed. Then, focusing on active power flow in the power system, the transient stability can be analyzed by the swing equation with periodic force which corresponds to the external power swing[1].

5.2 Numerical Results

Based on the practical system, the numerical simulation is performed for the following parameters [10]:

$$\begin{gathered} b = 0.7, \quad p_{\rm m} - p_{\rm e(dc)} = 0.2, \quad \varepsilon = 0.1, \\ D = 0.5 \quad \text{and} \quad \Omega = 0.05. \end{gathered} \tag{30}$$

Figure 4 shows the stability region, original separatrix and analytical criterion by the proposed method for the autonomous swing equation with $a = 0$. The *black* line shows the original separatrix and the *white* line the proposed criterion. In the figure, the region is colored *light-gray* for normal operation, *dark-gray* for stepping out. Needless to say, the separatrix, which corresponds to the stability limit based on the direct method, becomes the sufficient condition for the stable operation in Fig. 4. Furthermore, the proposed criterion is apparently close to the stable manifold which corresponds to the stability boundary compared with the original separatrix.

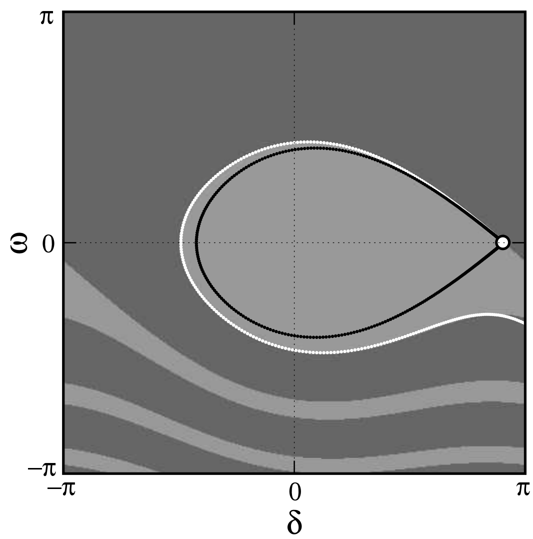
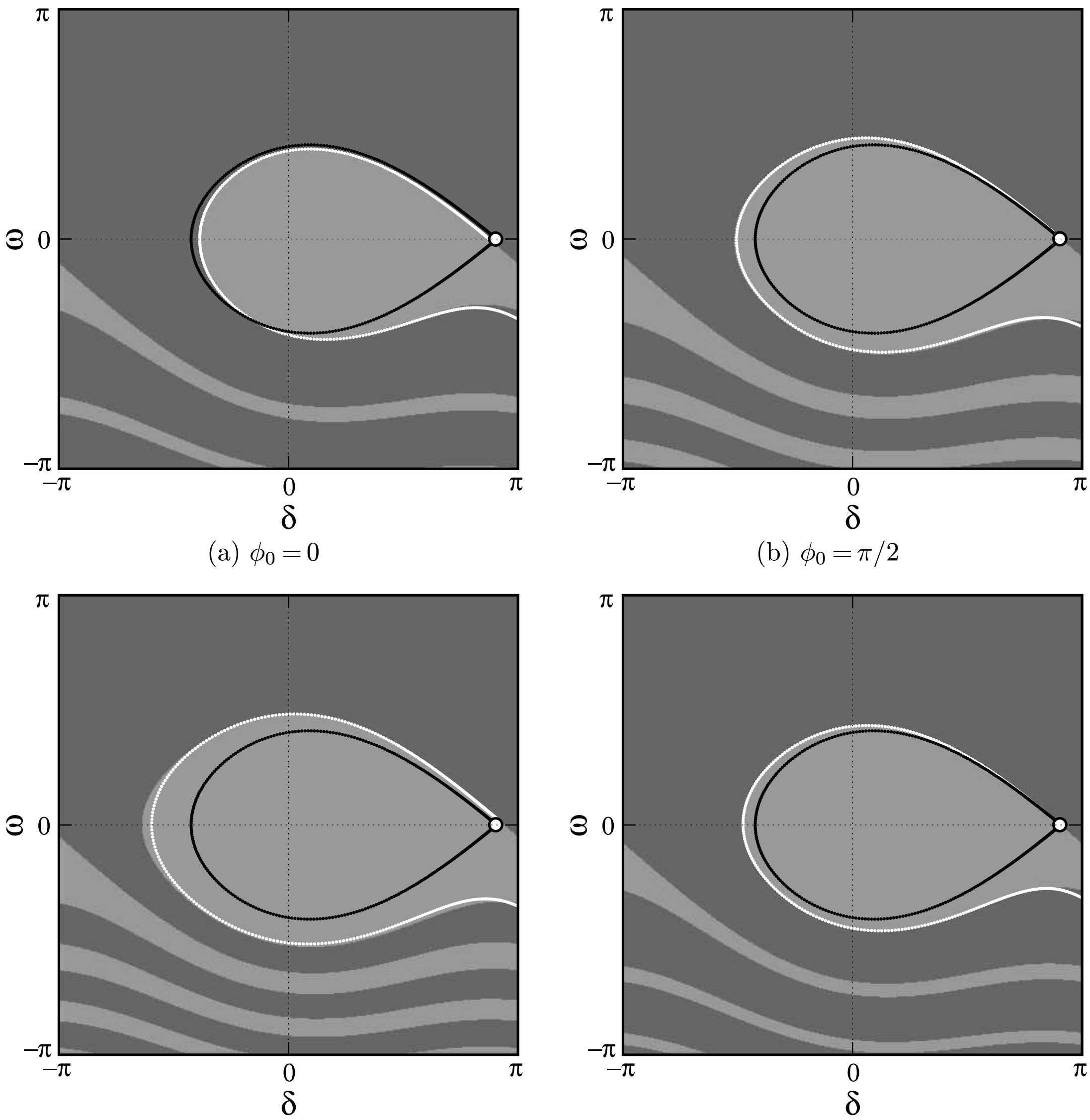


Fig. 4 Stability region, original separatrix and analytical criterion by the proposed method for the autonomous swing equation. The symbol ◯ denotes the assumed saddle type equilibrium point.

Figure 5 displays the stability regions, original separatrices and analytical criteria by the proposed method at the stroboscopic phase $\phi_0 = k\pi/2$ ($k = 0, \ldots, 3$) for the non-autonomous swing equation with $a = 0.7$. In the figures, the regions are colored *light-gray*

[1] The strict model of the ac/dc system should be based on a differential-algebraic equation [17,18].



(a) $\phi_0 = 0$ (b) $\phi_0 = \pi/2$

(c) $\phi_0 = \pi$ (d) $\phi_0 = 3\pi/2$

Fig. 5 Stability regions, original separatrices and analytical criteria by the proposed method at the stroboscopic phase $\phi_0 = k\pi/2$ $(k = 0, \dots, 3)$ for the non-autonomous swing equation. The symbol ◯ denotes the assumed saddle type equilibrium point.

for normal (slightly swing) operation. The rest regions and lines are drawn in the same way as in Fig. 4. In Fig. 5, the each criterion compasses almost perfectly stability region. These results make it clear that our proposed criterion is obviously much more effective than the classical Lyapunov's direct method.

6. Conclusions

In this paper, an analytical criterion for stability boundaries of non-autonomous systems is proposed. In particular, the method for the definition of the criterion is developed based on Melnikov's perturbation method. We have shown that the proposed criterion is also applied to an electric power system with dc transmission under periodic swing. The criterion is obviously much more effective than the Lyapunov's direct method. It can be applied not only to autonomous systems but also to non-autonomous systems.

The proposed method has many possibilities to expand itself to various non-autonomous systems. In the method, we adopt the relation between the separatrix and the stable manifold in order to evaluate the stability boundary. It can be thus modified itself to the system which has complicated stability boundaries, for examples, fractal basin boundaries. In addition, if Melnikov's methods for the higher degree of freedom systems [19] are considered, the proposed method can also be generalized without special formulation. The generalization is being prepared as a forthcoming paper.

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Appendix

Proof of Lemma 3

Applying the Gronwall's inequality [13,14] to the basic systems (2) and (6), the following relationship is given based on **Lemma 2**:

$$||\boldsymbol{q}_\varepsilon^{\mathrm{s}}(t)-\boldsymbol{q}_0(t-t_0)|| = \mathcal{O}(\varepsilon) \quad \text{for} \quad 0 \le t < +\infty. \tag{A1}$$

Using Eq. (2), $\boldsymbol{q}_\varepsilon^{\mathrm{s}}(t)$ satisfies

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{q}_\varepsilon^{\mathrm{s}}(t) = \mathrm{J}\mathrm{D}H(\boldsymbol{q}_\varepsilon^{\mathrm{s}}(t)) + \varepsilon \boldsymbol{g}(\boldsymbol{q}_\varepsilon^{\mathrm{s}}(t), \Omega t+\phi_0). \tag{A2}$$

Since $\boldsymbol{q}_\varepsilon^{\mathrm{s}}(t)$ is of class C^r for ε and t, Eq. (A2) can be differentiated with respect to ε. The interchange of the differential order by ε and t is obviously possible. Then, differentiating (A2) with respect to ε and interchanging the differential order by ε and t, the following formula is obtained:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial}{\partial\varepsilon}\boldsymbol{q}_\varepsilon^{\mathrm{s}}(t)\right) &= \mathrm{J}\mathrm{D}^2H(\boldsymbol{q}_\varepsilon^{\mathrm{s}}(t))\frac{\partial}{\partial\varepsilon}\boldsymbol{q}_\varepsilon^{\mathrm{s}}(t)\\ &\quad + \boldsymbol{g}(\boldsymbol{q}_\varepsilon^{\mathrm{s}}(t), \Omega t+\phi_0) + \mathcal{O}(\varepsilon).\end{aligned} \tag{A3}$$

For $\varepsilon=0$, using Eq. (A1), this lemma holds.

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薄　良彦 (学生会員)

1977年12月31日生．2000年京都大学工学部電気電子工学科卒業．2002年京都大学大学院工学研究科電気工学専攻修士課程修了, 同年同専攻博士後期課程に進学し, 現在に至る．直流送電を含む電力系統の動的挙動と制御に関する研究に従事．電気学会, IEEEの各会員．

引原　隆士 (正会員)

1958年8月9日生．1987年3月京都大学大学院工学研究科電気工学専攻博士後期課程研究指導認定退学．同年4月関西大学工学部助手．同専任講師, 助教授を経て, 1997年4月京都大学大学院工学研究科電気工学専攻助教授, 2001年8月同教授となり, 現在に至る．主として, 磁気浮上システムの開発, パワーエレクトロニクス, 非線形現象の解析および工学的応用, カオス制御等の研究に従事．電気学会, 電子情報通信学会, IEEE, APS等の各会員．